

Dual Nature of Radiation & Matter

This chapter is where light and matter trade identities. The photoelectric effect shows that light — normally explained as a wave through reflection, refraction, interference and diffraction — must sometimes be treated as a stream of discrete energy packets (photons) to explain what's actually observed. De Broglie then flips the idea around: if light, a wave, can behave like particles, perhaps particles like electrons can behave like waves too. The Davisson–Germer experiment confirms exactly that, and along the way this dual wave–particle picture quietly explains why Bohr's orbits had to be quantised in the first place.

1. The Photoelectric Effect: Discovery and Basic Phenomenon

The photoelectric effect is the ejection of electrons from a metal surface when light of suitable frequency falls on it. The electrons ejected are called photoelectrons, and the resulting current is photoelectric current.

- Hertz first discovered the effect (while studying electromagnetic waves); Lenard carried out the detailed experimental study and established its laws; Einstein supplied the theoretical explanation using the quantum theory of light, for which he won the Nobel Prize.
- Hertz's experiment: ultraviolet rays falling on the negative plate of an evacuated quartz-tube discharge gap make conduction through the tube noticeably easier.
- Hallwachs' experiment: a negatively charged zinc plate of an electroscope, when illuminated with ultraviolet light, loses its negative charge, becomes neutral, and then turns positive — direct evidence that negative particles are being emitted from the metal under illumination.
- Lenard's experiment: UV light incident on the cathode of a vacuum tube ejects electrons that are attracted to the anode, completing the circuit and producing a measurable photocurrent; UV light incident on the anode instead ejects electrons too, but no current flows (there's nothing on the other side to attract them across the gap).

2. Types of Electron Emission and Work Function

Electrons can be freed from a metal surface in several distinct ways — photoelectric emission is just one of (at least) three standard mechanisms, each requiring the electron to overcome the same underlying barrier: the work function.

- Work function (ϕ_0): the minimum energy an electron needs to escape from a given metal's surface; it depends on the nature of the metal and the condition of its surface.
- Thermionic emission: heating the metal supplies free electrons with enough thermal energy to escape (used in vacuum-tube cathodes).
- Field emission: an extremely strong electric field ($\sim 10^8$ V/m) pulls electrons directly out of the metal (as in a spark plug).
- Photoelectric emission: light of suitable frequency striking the surface ejects electrons directly — short-wavelength (high-frequency) light is more effective at this than long-wavelength (low-frequency) light.

3. Quantum Theory: Energy, Momentum and Effective Mass of a Photon

Planck's quantum hypothesis treats light as travelling in discrete energy packets called photons rather than as a continuous wave — and each photon, despite being massless at rest, carries definite energy, momentum and an associated 'effective mass' while in motion.

Energy of a photon

$$E = h\nu = hc/\lambda = 12400/\lambda(\text{in } \text{\AA}) \text{ eV} [hc = 12400 \text{ eV}\cdot\text{\AA}]$$

Linear momentum of a photon

$$p = E/c = h/\lambda$$

Effective mass of a photon (in motion)

$$m = E/c^2 = h/(c\lambda) \implies m \propto 1/\lambda$$

- Energy is directly proportional to frequency ($E \propto \nu$) — Planck's central postulate — and inversely proportional to wavelength.
- Since $m \propto 1/\lambda$, a violet-light photon (shorter λ) has greater effective mass than a red-light photon (longer λ); rest mass of a photon is always exactly zero — the 'effective mass' only exists because the photon is in motion.
- $hc = 12400 \text{ eV}\cdot\text{\AA}$ is a constant worth memorising outright — it turns every photon-energy calculation into pure arithmetic once λ is known in ångströms.

4. Intensity of Light and Photon Flux

Intensity connects the everyday idea of how 'bright' a light source is to the underlying photon picture: more intensity simply means more photons crossing a given area every second.

Intensity

$$I = P/A = N(h\nu)/(At) = n(h\nu)/A [n = \text{photons/second}]$$

Photon flux from intensity

$$n = P/(h\nu) = P\lambda/(hc) \approx (5 \times 10^{24} \text{ J}^{-1}\text{m}^{-1}) \times P \times \lambda$$

- Intensity has SI unit W/m^2 (equivalently $\text{J}\cdot\text{m}^{-2}\cdot\text{s}^{-1}$) and is directly proportional to the number of photons arriving per second.
- Intensity $\propto$ photons/second $\propto$ photoelectrons/second $\propto$ photocurrent $\propto 1/d^2$ for a point source at distance d — this inverse-square chain is the single fact behind every 'distance changed' numerical in this topic.

5. Radiation Force and Radiation Pressure

Because photons carry momentum, light striking a surface transfers momentum to it — exerting a measurable (if tiny) force and pressure, with the result depending on whether the surface reflects or absorbs the light.

Perfectly reflecting surface, normal incidence

$$F = 2P/c ; \text{Pressure} = 2I/c$$

Perfectly absorbing surface, normal incidence

$$F = P/c ; \text{Pressure} = I/c$$

- A reflecting surface experiences exactly double the force of an absorbing surface for the same incident power — the photon's momentum reverses direction on reflection ($\Delta p = 2h/\lambda$ per photon) instead of simply vanishing ($\Delta p = h/\lambda$ per photon) on absorption.
- These results follow directly from $\Delta p(\text{total})/\Delta t$ for N photons striking per second, combined with $n = P/(hc/\lambda)$ for the photon flux.

6. Lenard's Experimental Study: Effect of Collector Potential

Lenard systematically varied the potential between the photo-emissive cathode (C) and the collecting anode (A) to map out exactly how the photocurrent responds — this single experiment is the source of nearly every qualitative law of the photoelectric effect.

Stopping potential and maximum kinetic energy

$$K_{\text{max}} = eV_0$$

- As the anode is made increasingly positive relative to the cathode, photocurrent rises and then flattens out at a maximum value called the saturation current — at this point essentially every photoelectron emitted is being collected.
- As the anode is made increasingly negative instead, photocurrent falls and eventually reaches zero at a particular negative potential called the stopping potential (V_0) — the minimum reverse potential needed to stop even the most energetic photoelectrons from reaching the anode.
- Because V_0 specifically measures the maximum kinetic energy of the photoelectrons (the easiest electrons to stop are the slow ones; the hardest are the fastest), $K_{\text{max}} = eV_0$ exactly.

7. Effect of Intensity and Effect of Frequency

Two of Lenard's most important — and most counter-intuitive, from a wave-theory standpoint — observations concern how the photocurrent graph shifts when intensity is changed versus when frequency is changed.

- Increasing the intensity of light (at fixed frequency) increases the saturation current i_s proportionally, but leaves the stopping potential V_0 completely unchanged — more light means more photoelectrons, not faster ones.
- Increasing the frequency of light (at fixed intensity) increases the stopping potential V_0 , while the saturation current stays essentially unchanged — higher frequency means each photoelectron carries more energy, not that there are more of them.
- Below a certain minimum frequency, no photoelectrons are emitted at all, no matter how intense the light — this minimum is the threshold (or cut-off) frequency ν_0 ; the corresponding maximum wavelength that can still eject electrons is the threshold wavelength λ_0 .

8. Threshold Frequency, Time Lag and Other Key Observations

A handful of supporting observations round out the experimental picture and are exactly the facts that any successful theory of the photoelectric effect has to explain.

- There is no measurable time lag between the light striking the surface and the photoelectrons being emitted (delay $\leq 10^{-9}$ s) — the process is effectively instantaneous, even for very dim light.
- Stopping potential does not depend on the distance between the light source and the photo-emissive surface (only the saturation current does, through intensity $\propto 1/d^2$).
- Ultraviolet light causes photoemission from virtually any metal surface; ordinary visible light causes photoemission only from alkali metals (Na, K, Cs, Rb), whose work functions are unusually low.

9. Failure of the Wave Theory of Light

Classical (wave) theory treats light's energy as spread continuously across the wavefront — and that single assumption runs straight into trouble on three separate fronts when checked against Lenard's observations.

- Time lag: if energy arrives continuously and spreads over the whole illuminated surface, an electron should need a finite accumulation time before it has enough energy to escape — but experimentally there is no such delay.
- Intensity vs. energy: wave theory predicts that a more intense beam should deliver more energy to each electron, making the emitted electrons faster (and raising V_0) — but V_0 is observed to be completely independent of intensity.
- Threshold frequency: wave theory has no reason to expect a cut-off frequency at all — given enough intensity (and hence enough time to accumulate energy), light of any frequency should eventually eject electrons. No such threshold should exist, yet one is always observed.

10. Einstein's Photoelectric Equation

Einstein resolved every one of the wave theory's failures with a single postulate: light energy doesn't spread out at all, but arrives in discrete quanta (photons), each of which can interact with only one electron at a time and transfers its energy in full, instantly.

Energy balance for one photon–electron interaction

$$h\nu = K_{\text{max}} + \phi_0 + Q \quad (Q = \text{energy lost in collisions before emission})$$

Einstein's photoelectric equation ($Q = 0$)

$$K_{\text{max}} = h\nu - \phi_0 \iff eV_0 = h\nu - h\nu_0$$

Threshold condition

$$h\nu_0 = \phi_0 \implies \phi_0 = hc/\lambda_0 = 12400/\lambda_0(\text{\AA}) \text{ eV}$$

- Because one photon transfers its entire energy to (at most) one electron, the process is instantaneous by construction — there's no accumulation time needed, immediately explaining the absence of any time lag.
- Increasing intensity increases the number of photons arriving per second, hence the number of photoelectrons emitted per second ($i_s \propto I$) — but each individual photon–electron interaction is unaffected, so K_{max} (and hence V_0) stays the same. This is exactly what's observed.
- Since K_{max} can never be negative, $h\nu$ must exceed ϕ_0 (equivalently $\nu > \nu_0$) for any emission to occur at all — this single inequality is the threshold frequency, derived rather than assumed.
- Einstein's equation is fundamentally a statement of conservation of energy — and his explanation, built entirely on Planck's quantum hypothesis, won him the 1921 Nobel Prize in Physics.

11. Graphs: K.E.max vs ν and Stopping Potential vs ν

Plotting Einstein's equation against frequency turns it into the equation of a straight line — and the slope of that line is one of the most famous fundamental constants in physics.

K.E.max vs ν (slope = h)

$$K_{\text{max}} = h\nu - \phi_0 \quad (\text{compare } y = mx - c)$$

Stopping potential vs ν (slope = h/e)

$$V_0 = (h/e)\nu - (\phi_0/e)$$

- Both graphs are straight lines whose slope is the same for every metal (h , or h/e respectively) — only the y-intercept ($-\phi_0$, or $-\phi_0/e$) differs from one metal to another, since work function depends on the material.
- A metal with a larger work function has a larger threshold frequency ν_0 (the x-intercept) and a more negative y-intercept, but exactly the same slope as any other metal — this universality of slope is itself strong evidence for the photon picture.
- These graphs are the basis of a classic experimental method for measuring Planck's constant: plot V_0 against ν for a given metal, and read off h from the slope (multiplying by e).

12. Quantum Efficiency and Photoelectric Current

Not every photon that strikes a photo-emissive surface succeeds in knocking an electron loose — quantum efficiency is simply the fraction that do, and it converts directly into the actual measured photocurrent.

Quantum efficiency

$$\eta = n_e/n_{ph} \text{ (} n_e = \text{electrons emitted/s, } n_{ph} = \text{photons incident/s)}$$

Photoelectric current

$$I_e = n_e \times e = 1.6 \times 10^{-19} \times n_e$$

- If quantum efficiency is given as x%, then $n_e = (x/100) \times n_{ph}$ — combine this with $n_{ph} = P\lambda/(hc)$ to get the number of photoelectrons per second directly from the beam's power and wavelength.
- Quantum efficiency depends on the material and the photon energy; it is always ≤ 1 (i.e., $\leq 100\%$), since a photon can free at most one electron.

13. Compton Effect and Raman Effect

Two further phenomena, alongside the photoelectric effect, independently confirm that light behaves like a stream of particles in certain interactions — both are scattering processes, but with subtly different mechanisms.

- Compton effect: high-energy photons (X-ray or γ -ray) scattering off free (or loosely bound) electrons lose some energy to the electron and emerge with a longer wavelength — direct evidence that photons carry momentum that obeys ordinary collision mechanics.
- Raman effect: monochromatic visible light scattering inelastically off the molecules of a transparent medium emerges with wavelength either increased or decreased, due to energy exchange with the molecules' vibrational/rotational states.
- Both effects, together with the photoelectric effect, form the experimental backbone for treating light as having genuine particle-like (quantum) character in matter–light interactions.

14. The Photocell

The photocell is the direct practical payoff of the photoelectric effect — a sealed device that converts incident light directly into a measurable electric current, with that current tracking the light intensity in real time.

- Construction: an evacuated, sealed glass (or quartz) tube containing an anode and a concave photo-emissive cathode (commonly caesium), so that emitted electrons are efficiently focused towards the anode.
- Working: light of frequency above the cathode's threshold ejects photoelectrons, which the anode collects, driving a current in the external circuit; the current rises with increasing light intensity and stops the instant illumination stops.
- Applications: television camera tubes, automatic door sensors, burglar alarms, and automatic switching of street lights and traffic signals — anywhere a light signal needs to be converted instantly into an electrical one.

15. Dual Nature of Light

Putting the wave-based phenomena (reflection, refraction, interference, diffraction) and the particle-based phenomena (photoelectric effect, Compton effect, pair production/annihilation) side by side leads to an inescapable conclusion about the nature of light itself.

- Light doesn't have one single, fixed nature — instead, which aspect (wave or particle) it displays depends entirely on which experiment you perform; this is the dual nature of light.
- Crucially, the two natures are never observed together in the same single experiment: an experiment either reveals wave behaviour or particle behaviour, never both simultaneously — a complementarity that carries over directly to matter waves as well.

16. de Broglie Hypothesis and Wavelength of a Moving Particle

If light — fundamentally a wave — can act like particles, de Broglie reasoned, then matter — fundamentally particles — might equally act like waves. The wavelength he associated with a moving material particle is now named after him.

de Broglie wavelength (general)

$$\lambda = h/p = h/(mv) = h/\sqrt{2mE}$$

- For macroscopic objects, λ works out to roughly 10^{-24} Å — far smaller than even γ -ray wavelengths ($\sim 10^{-5}$ Å, the smallest measurable scale), which is exactly why everyday objects never show observable wave behaviour.
- For microscopic particles (electrons, protons, neutrons, atoms), λ comes out around 10^{-10} m — comparable to X-ray wavelengths — squarely within measurable range, which is why matter waves are observable for these particles but not for cricket balls or cars.

17. de Broglie Wavelength of Charged and Thermal (Uncharged) Particles

For a charged particle accelerated through a known potential difference, or an uncharged particle in thermal equilibrium, the general de Broglie relation specialises into two extremely useful working formulas.

Charged particle accelerated through V

$$\lambda = h/\sqrt{2mqV}$$

Electron specifically (in Å, V in volts)

$$\lambda = 12.27/\sqrt{V} \text{ Å} \implies \lambda \propto 1/\sqrt{V}$$

Voltage to stop an electron of wavelength λ

$$V = 150.6/\lambda^2 \text{ (in Å) volts}$$

Thermal (uncharged) particle at temperature T

$$\lambda = h/\sqrt{(3mkT)} \text{ [from } E = (3/2)kT]$$

- The $12.27/\sqrt{V}$ formula for electrons is worth memorising directly — it converts an accelerating voltage straight into a wavelength in ångströms without needing to plug through h , m and q each time.
- $\lambda \propto 1/\sqrt{V}$ means quadrupling the accelerating potential only halves the de Broglie wavelength — a very common source of numerical traps when problems ask 'by what factor does λ change'.
- For uncharged particles like neutrons or gas molecules, thermal kinetic energy $(3/2)kT$ plays the same role that qV plays for charged particles accelerated through a potential difference.

18. Davisson–Germer Experiment

The Davisson–Germer experiment was the first direct experimental confirmation of de Broglie's matter-wave hypothesis, showing that a beam of electrons diffracts off a crystal lattice exactly the way a wave would.

Bragg's equation (as applied to the crystal planes)

$$2d \sin\theta = n\lambda$$

Diffraction-angle form

$$D \sin\phi = n\lambda$$

Relation between glancing angle θ and diffraction angle ϕ

$$\phi = 180^\circ - 2\theta \iff \theta = 90^\circ - \phi/2$$

- Apparatus: an electron gun produces electrons of known, adjustable energy (via thermionic emission and acceleration); these strike a nickel crystal, which acts as a three-dimensional diffraction grating; a detector measures the intensity of electrons scattered at each angle.
- Result: at an accelerating voltage of 54 V, the scattered-electron intensity shows a sharp maximum at a diffraction angle of 50° .
- Using the diffraction geometry (Bragg-type analysis), the experimentally measured electron wavelength comes out to 1.65 Å, while de Broglie's formula $\lambda = 12.27/\sqrt{V}$ predicts 1.66 Å for 54 V electrons — the near-exact agreement was the decisive confirmation that moving electrons genuinely behave as waves.
- d in Bragg's equation is the spacing between successive crystal planes (interplanar spacing); D is the spacing between atoms within a single lattice plane; n is the diffraction order; θ is the glancing angle and ϕ the diffraction angle.

19. Explanation of Bohr's Quantisation Condition

One of the most satisfying payoffs of the de Broglie picture is that it explains, rather than just assumes, why Bohr's electron orbits had to have quantised angular momentum in the first place.

Standing-wave condition for a stable orbit

$$2\pi r = n\lambda$$

Substituting $\lambda = h/(mv)$

$$mvr = nh/(2\pi) \text{ — Bohr's quantisation condition}$$

- De Broglie pictured the electron in orbit as a standing (stationary) matter wave wrapped around the nucleus — exactly like a vibrating string whose two ends meet seamlessly. This only works without destructive self-interference if the orbit's circumference is an exact integer multiple of the electron's own de Broglie wavelength.
- Equivalently, picture the circular orbit 'unrolled' into a straight line of length $2\pi r$: for the wave to join up consistently when re-rolled into a circle, that length must contain a whole number of wavelengths — precisely $2\pi r = n\lambda$.
- Substituting the de Broglie wavelength directly produces Bohr's originally postulated (but unexplained) quantisation rule $mvr = nh/2\pi$ — turning what was an ad-hoc assumption in the Bohr model into a natural consequence of wave mechanics.

20. Wave Packets, Wave Function and the Quantum View

A few deeper conceptual points round out the picture of matter waves — connecting the de Broglie wavelength to the uncertainty principle, and introducing the wave function that genuinely describes a quantum particle.

- A particle with perfectly definite momentum ($\Delta p = 0$) has one single, exactly defined wavelength $\lambda = h/p$ — but such a wave extends through all of space ($\Delta x \rightarrow \infty$), meaning the particle's position is completely undetermined. This is consistent with, and in fact required by, the uncertainty principle.
- To describe a particle actually localised in some finite region (Δx finite), many waves of slightly different wavelengths (centred on h/p) must be superposed together — the resulting localised disturbance is called a wave packet, and it necessarily has Δp finite (not zero) as well.
- The matter wave is described by a (generally complex/imaginary-valued) wave function ψ ; Max Born's interpretation is that $|\psi|^2 (\Delta V)$ gives the probability of finding the particle within a small volume ΔV — ψ itself carries no direct physical meaning, only $|\psi|^2$ does.
- The de Broglie wavelength $\lambda = h/p$ has genuine physical significance, but the associated phase velocity of the matter wave does not directly correspond to anything observable; it is the group velocity of the wave packet that is physically meaningful and equals the actual velocity of the particle.
- Photoelectric-effect observations show that energy is absorbed in discrete units of $h\nu$ during light-matter interaction — a more cautious and precise statement than simply 'light is made of particles', since it describes only how energy transfers, not necessarily what light fundamentally 'is'.
- The independence of stopping potential from intensity, together with its clean dependence on frequency, is the single most decisive experimental fact separating the photon picture from the classical wave picture of light.