

Kinematics

Kinematics describes motion using position, velocity, and acceleration, without asking what causes it. From the basic equations of motion through projectile motion, relative velocity, and river-boat problems, everything in this chapter builds on one idea: motion can be described completely if we know how position changes with time.

1. Frame of Reference and Position

Motion is always described relative to a frame of reference — a fixed point or object from which position is measured. Without a reference frame, the question "is this object moving?" has no meaningful answer.

- Position is the location of an object relative to a chosen origin.
- Displacement is the change in position — a vector quantity with both magnitude and direction.
- Distance is the total path length covered — a scalar quantity, always $\geq$ displacement magnitude.

2. Speed and Velocity

Speed tells you how fast something moves; velocity tells you how fast and in which direction.

AVERAGE VELOCITY

$$v_{\text{avg}} = \Delta x / \Delta t$$

INSTANTANEOUS VELOCITY

$$v = dx/dt$$

- Average speed = total distance / total time
- Average velocity = total displacement / total time
- Instantaneous velocity is the velocity at a specific moment — found by taking the derivative of position with respect to time.

3. Acceleration

Acceleration is the rate of change of velocity. It does not require speed to be increasing — slowing down or changing direction also counts as acceleration.

AVERAGE ACCELERATION

$$a_{\text{avg}} = \Delta v / \Delta t$$

INSTANTANEOUS ACCELERATION

$$a = dv/dt = d^2x/dt^2$$

4. Equations of Motion (Constant Acceleration)

These three equations apply only when acceleration is constant. They are the single most exam-relevant result in this chapter — almost every numerical in JEE and NEET kinematics traces back to one of these three.

FIRST EQUATION

$$v = u + at$$

SECOND EQUATION

$$s = ut + \frac{1}{2}at^2$$

THIRD EQUATION

$$v^2 = u^2 + 2as$$

- u = initial velocity, v = final velocity, a = acceleration, t = time, s = displacement.
- Sign convention matters: pick one direction as positive and stay consistent for the entire problem.

5. Motion Under Gravity

Free fall is a special case of constant acceleration motion, where $a = g \approx 9.8 \text{ m/s}^2$, directed downward.

- For an object dropped from rest: $u = 0$, and the equations of motion apply directly with $a = g$.
- For an object thrown upward: take upward as positive, so $a = -g$ during the rise and fall.
- Time to reach maximum height: $t = u/g$. Maximum height: $h = u^2/2g$.

6. Relative Velocity

The velocity of one object as observed from another moving object — found by vector subtraction.

$$\text{RELATIVE VELOCITY OF A WITH RESPECT TO B} \quad v_{AB} = v_A - v_B$$

- Common JEE/NEET application: rain falling on a moving observer, or two trains/cars approaching or moving apart.

7. Projectile Motion

A projectile undergoes two independent motions simultaneously: uniform horizontal velocity, and vertically accelerated motion under gravity. These two motions don't affect each other — this independence is the key insight of the entire topic.

TIME OF FLIGHT	$T = 2u \sin\theta / g$
MAXIMUM HEIGHT	$H = u^2 \sin^2\theta / 2g$
HORIZONTAL RANGE	$R = u^2 \sin(2\theta) / g$

- Range is maximum when $\theta = 45^\circ$.
- At the highest point, vertical velocity is zero, but horizontal velocity remains $u \cos\theta$ throughout the flight.

8. Horizontal Projection from a Height

When a body is launched horizontally from some height h , it has no initial vertical velocity. The fall happens purely due to gravity while the horizontal motion continues undisturbed at constant speed.

TIME TO REACH THE GROUND	$t = \sqrt{2h/g}$
HORIZONTAL RANGE COVERED	$R = u\sqrt{2h/g}$
SPEED ON LANDING	$v = \sqrt{u^2 + 2gh}$

- The time to fall depends only on the height h , not on the horizontal speed u .
- This is the same idea Galileo used to argue that a bullet fired horizontally and a bullet simply dropped hit the ground at the same instant.

9. Oblique Projection from a Height

This is the general case: a body is launched at an angle θ from a height h , either above or below the horizontal. The vertical motion now has an initial component too.

- If launched at angle θ above the horizontal from height h , the body first rises, then falls the full height plus the extra rise before reaching the ground.
- If launched at angle θ below the horizontal, both gravity and the initial velocity pull the body downward together, so it reaches the ground sooner.
- Break the velocity into horizontal ($u \cos\theta$) and vertical ($u \sin\theta$) components first, then handle each direction separately before combining.

10. Rain-Man Problems (Relative Velocity in 2D)

Rain may be falling straight down, but a person walking forward feels it hitting them at an angle — that apparent angle is the direction of the rain's velocity relative to the walker.

VELOCITY OF RAIN RELATIVE TO MAN

$$\mathbf{v}_{RM} = \mathbf{v}_R - \mathbf{v}_M$$

- Draw the rain's and the man's velocity as vectors, then subtract them as vectors to get the relative velocity.
- The angle the umbrella must be tilted equals the angle between the relative velocity vector and the vertical.

11. River-Boat Problems

A boat's velocity relative to the ground is the sum of its velocity relative to the water and the river's own velocity.

BOAT'S VELOCITY RELATIVE TO GROUND

$$\mathbf{v}_{\text{boat}} = \mathbf{v}_{\text{boat/water}} + \mathbf{v}_{\text{river}}$$

- Shortest path (land directly opposite): angle the boat partly upstream to cancel the river's sideways push — takes longer.
- Shortest time: point straight across; the current carries the boat downstream, but the crossing itself is fastest.